Mathematics Education and Technology is a fast evolving domain. In 2005, the 17th ICMI Study discussion document\(^1\) stressed (p.12) the “potential and challenges for mathematics education of the increasing levels of connectivity”, and proposed to address issues like the potential contribution to mathematics learning of different levels of interactivity and different modalities of interaction, the potential of collaborative study of mathematics whilst physically separated and the potential for creating virtual communities for mathematics learning. Teaching mathematics online was still uncommon, and the few contributions presented to the study and included in the book mostly deal with experiments of online collaboration between distant classrooms (Hoyles et al., 2010). Even the chapter about Automatic Assessment supported by Digital Technologies (Sangwin, Cazes, Lee & Wong, 2010) does not directly refer to the usability of this modality of assessment for online courses.

Today, nine years later, the layman becomes aware of e-learning because of the emergence of systems with unlimited numbers of participants under the denomination of Massive Open Online Courses (MOOC) or of freely distributed video lessons on the web (online academies).

What is less well known is that these applications build on nearly twenty years of small, medium or large scale implementations of distance teaching/learning practices which increasingly used the Internet. Especially with regard to mathematics education, these practices are not well documented or investigated, and there is a risk that they will remain invisible behind massive online applications.

Research is needed to investigate the mathematics teaching/learning practices that online applications can promote or help, the potential of these practices, and how it actualises in real settings; more generally, to investigate how the emergence of these practices changes the landscape of Mathematics Education at university level. This is why the book, *Teaching Mathematics Online: Emergent Technologies and Methodologies*, is particularly well-timed.

The book can be seen as a collection of practices and evaluations involving a great variety of audiences, types of interaction and learning objectives. I will try to illustrate this variety through a brief review of the chapters in each section, and then offer my own reflection with regard to the “state of the art” as it appears in the book.

The book is divided into three sections. Chapters in the first two sections report on and analyse practical implementation of e-Learning courses or resources. The editors distinguish between “blended” and “pure online” experiences. In “blended experiences”, the focus of section one, online applications supplement existing face-to-face courses. In Chapter 1, the course is for elementary mathematics teachers. Online discussion supplements regular teaching in order to promote a reflective approach to mathematical notions. In the other section one chapters, the courses are for college or university students. Authors insist on current challenges experienced in mathematics courses, especially with regard to students’

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motivation and preparation. In Chapter 2 online assignments are proposed, in order to make engineering students more active and to promote student-instructor interaction. In Chapter 3, “screen casts”, that is to say short videos, are offered both on-campus and at a distance. The videos show live lectures, as well as responses to students’ enquiries and short explanations of critical topics.

Chapters 4 and 5 introduce Learning Management Systems (LMS), that is to say Web platforms offering facilities for administrating, documenting and monitoring courses as well as delivering organised learning contents. Chapter 4 presents the use of LMS like Intelligent Web Teacher (IWT) and Modular Object Oriented Dynamic Learning Environment (Moodle) and the various tools and opportunities that these systems provide: identification of learning objects and learning activities, as well as self and peer assessment. An implementation for a freshmen geometry course is discussed. The author of Chapter 5 reports on her experience of using various web-based tools along with a LMS to implement “hybrid” (another word for “blended”) courses for her mathematical college and university classes. She uses course announcement and discussion boards, videos and collaborative authorship.

Closing this first section, Chapter 6 presents various strategies in “blended e-learning” to deal with current challenges experienced in mathematics courses for ICT students: heterogeneity of students, low level of pre-knowledge, motivation and achievements, and high drop-out rates of particular groups of students.

Section 2 focuses on “Pure online experiences in Mathematics e-Learning”. Chapters 8 and 11 explore the potential of online multimedia learning materials to deal with the difficult transition from secondary to higher education. Chapter 8 describes how online applications can help a program of “Bridging Education”. The program is based on the administration of an entry test and the organisation of an online summer course. In Chapter 11, a particular focus is put (1) on the design of interactive learning materials by way of an authoring system and (2) on the design and evaluation of learning course scenarios, one privileging attendance and the other more e-learning oriented. Chapter 12 analyses and compares mathematics e-learning courses in four universities around the world. These universities offer up to 4,000 online courses in mathematics, with an attendance reaching 100,000 students. In addition to data characterising each university, the chapter compares the mathematics e-learning courses from three different perspectives: the virtual learning environment used, the teaching/learning methodology, and the main challenges addressed and factors of success.

The other chapters in section 2 deal with distance adult education, in particular in-service teacher professional development. Chapter 7 deals with teachers’ “online communities of practice”. The author explores how the use of web-based technologies can promote the creation of online communities that share knowledge and experiences regarding mathematical content and classroom practice. In particular, she analyses an online learning experience regarding a multinational group of elementary and middle-school teachers of Statistics. Chapter 9 deals with adult education in mathematics. The author discusses the use of various software and platforms as well as instructional strategies for efficient online learning. Chapter 10 is about in-service mathematics teachers taking an online Master’s course in mathematics education with a special focus on asynchronous collaboration and reflective conversation. Three case studies show how teachers progress towards a better understanding of mathematical notions and of their teaching.

Contrasting with specific experiences studied in the first two sections, the chapters in section 3 focus on tools and resources for mathematics e-learning. Among these, high ranking websites for collegiate mathematics in the USA are conspicuous resources. Chapter 14 examines and classifies data issued from quantam.com, and raises questions about who visits these websites and in what circumstances. The websites are also analysed from different
points of view: their interactivity and dynamic capabilities, and their pedagogical and mathematical relevance.

Chapters 15 and 16 adopt a design perspective with regard to LMS. Chapter 15 reports on the development of a system based on mathematics learning blocks and tools for easy creation of online mathematics content. A practical example illustrates the flexibility and potential of the system. Chapter 16 presents a mathematics teaching experience based on the combination of an e-learning platform and a number of mathematical pieces of software for mathematics refresher courses.

Also adopting a design perspective, Chapters 13 and 17 deal with particular components of online courses. In the vein of Sangwin et al. (2010), Chapter 13 explores the affordance and constraints of computer-aided assessments as a component of mathematics online courses. In particular, there is a need for automatically evaluating whether or not two systems of equations are equivalent. The implementation of computer algebra algorithms is the solution. Chapter 17 reports on an experience involving online communication between learners via an e-learning platform. It compares a formula editor existing on the platform with the use of direct scanner-based handwriting, and shows how handwriting continues to be a relevant way of communicating mathematics in e-learning programmes.

Closing the section and the book, Chapter 18 presents and discusses experiences of the development of using online learning materials to address issues faced by incoming “at-risk” students. The materials are based on three technologies: “touch-screen” to post on line notes written during class, podcasts recorded to supplement existing online resources, and screencasts (see Chapter 3 above) explaining critical excerpts of written materials.

This survey of chapters provides insight into the diversity of practices and models of practice involving online applications. This great diversity contrasts with a uniform view of online teaching/learning originating from the emergence of MOOCs and online “academies”. Somebody said that MOOCs are like large cruise ships, unvaryingly carrying passengers along the same sea routes. In some sense the experiences reported in the book are like a flotilla of different ships, some very small and some larger, offering flexibility to adapt for a variety of challenges concerning mathematics teaching/learning at university level.

There are challenges related to the students, particularly concerning heterogeneity, low level of pre-knowledge, motivation and achievements. Challenges concern the teachers, and adults in general, who need a reflective approach to mathematics in their profession. Challenges are also related to constraints and affordances of those digital technologies that are especially attached to mathematical activity. If we consider mathematical activity as work on a diversity of material or mental representations, it is important to evaluate how, in a given practice, technology actually provides representations with adequate affordances and feedback. Dynamicity is fundamental. The ability of an online learning system to react mathematically by way of embedded computer algebra is also important. Another specificity of mathematics is its particular system of written notation. Created as a very efficient tool for mathematical activity over centuries, notation seems very resistant to the recent explosion of pure text communication via computer networks, generating non-obvious problems for online conversations about mathematical notions. And finally, since one does not learn mathematics alone, flexible ways of doing mathematics and communicating together, even at a distance, are necessary.

In my view, there is at present no best way online technology can address these many challenges, and, while the Preface sets the goal to “identify and publish worldwide best practices regarding mathematics e-learning in higher education” (p. xi), I wonder whether such best practices emerge from the diversity presented here. For me, this is not a failure, but rather an encouragement to use this wealth of examples, analyses and evaluations to continue to experiment and evaluate a wide range of models.
I do not think either, that “insight and understanding into current and future trends regarding how mathematics instruction is being facilitated and leveraged with Web-based and other emerging technologies” (p. xi) really emerge from this rich collection of practices and evaluations. This would actually be a difficult task, and the structure of the book in three sections reflects the difficulty in identifying strong common dimensions in practice. Again, this is not a failure, but the recognition that, although uses are developing fast, research needs time and tools to reflect on practices. Many of the book’s chapters are based on the author’s or authors’ experience of setting up online courses or resources in their university. On the one hand, this ensures genuine reports about the successes and difficulties encountered in these experiences as well as the authors’ efforts. On the other hand, there is a dispersion of perspectives and methods of design and evaluation. Authors sometimes mention theories of teaching/learning, “variation theory”, for instance (pp.1–4, 7–9, 21) as an overarching framework rather than as an operational tool, orienting design and evaluation.

Building on these experiences, what we need is to turn this rich field of experiment into real didactical design, “that is to say the design of learning environments and teaching sequences informed by close analysis of the specific topic of concern and its framing within a particular subject area” (Ruthven et al., 2009, p. 329). Didactical design promotes intermediate theories and design tools mediating between “grand theories” and the practice of mathematics teaching. This is consistent with Cobb et al. (2003, p.11):

Design experiments tend to emphasise an intermediate theoretical scope that is located between a narrow account of a specific system and a broad account that does not orient design to particular contingencies.

By bringing together researchers around key questions posed by the design and use of digital environments, the ICMI book Technology and Mathematics Education (Hoyles & Lagrange, 2010) helped to focus on this intermediate level between particular experiences and grand theories. Frameworks like the instrumental approach, the theory of semiotic mediation or Valsiner’s zone theory, emerged as tools for communicating between researchers and for clarifying the conditions under which technology can contribute to students’ learning. A paper like Maracci, Cazes, Vandebrouck & Mariotti (2013) shows how, in the long-term, this tight connection between a specific topic of concern, relevant frameworks, and the design of learning environments and teaching sequences, pays off, when dealing with innovative use of technology in mathematics education.

As I said in the introduction, the landscape has evolved fast since the publication of the ICMI book. Reading the book under review, I am sure that if a similar ICMI study of the domain of mathematics e-learning was carried out now, there would be a wealth of experiments and theoretical elaborations to compare and analyse critically.

References


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